

# ENERGY COST AND OPTIMIZATION OF A CLOSED CIRCUIT CRUSHING PLANT WITH A CENTRIFUGAL CRUSHER

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## Abstract

A closed circuit crushing consists of a crusher that breaks the feed material, a screen that classifies the material and a conveyor system that returns the over size back to the crusher.

The capacity of a centrifugal crusher with a given motor power depends on its rotation frequency. The lower the rotation frequency the lower the specific energy, or breakage intensity, that the particles obtain before breakage and hence the greater the capacity of the crusher to provide a low specific energy to the feed particles

On the other hand, according to the breakage model the fraction of the feed that breaks below the feed size class is greater as the kinetic energy given to the particles increases. The combination of these two parameters has two effects:

- a) the energy cost per mass of the material broken by the crusher decreases as the breakage intensity decreases and
- b) the circulating load that has to be returned to the crusher increases as the breakage intensity decreases.

This paper shows how to calculate the optimum crusher breakage intensity, or rotation frequency, in order to minimize the energy cost per ton of the product broken below size.

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**Keywords:** Centrifugal crusher, energy cost, crushing circuit optimization

## Introduction

In crushing circuits the feed material to the crusher is not broken below the desired size in one pass and for this reason the product is usually screened. The coarse fraction returns to the crusher through a system of conveyor belts. The energy consumed by the crusher in each pass depends on the feed rate ton/h and the specific energy kWh/ton that is provided to the material during breakage, (breakage intensity). In general the higher the breakage intensity the finer the product obtained.

The method to control the breakage energy per ton of the material depends on the type of the crusher. In jaw crushers for instance, the breakage energy is controlled by the discharge opening and it is higher as the opening is smaller. This can be explained as follows. The motor power is given, the residence time in the crusher increases as the discharge opening is reduced and the feed rate drops accordingly, so the energy per unit mass of the material (breakage energy) increases. On the other hand the breakage energy can be reduced by increasing the discharge opening.

In centrifugal crushers, which are examined in the present work, the provided breakage energy depends on the rotation frequency of the crusher disk. The higher the rotation frequency the higher the breakage intensity provided to each particle. In opposition to the jaw crushers there is no restriction to the feed rate due to the discharge opening and the

parameters that control the feed rate are the motor power and the velocity of the particles in the crusher that determines the residence time. For a given motor power the higher rotation frequency gives higher breakage intensity and consequently the capacity drops. At low frequencies and low breakage energies the crusher capacity increases and the limit is imposed by the residence time that decreases with decreasing rotation frequency.

Different materials have different strengths and require smaller or higher breakage intensity. For the same material the higher the breakage intensity provided by the crusher the finer the size of the product obtained. In crushing circuits all the material is required to break below a given size and this is achieved by classification of the product on a screen. The passing fraction is the final product while the coarse fraction is recycled to the crusher that receives the fresh feed as well. The higher the quantity of the recycled material, the higher the quantity to which the crusher has to provide the breakage intensity. It can be understood now that the higher the breakage intensity the lower the feed rate but since the product is finer the recirculation of the coarse size will be reduced.

From the point of special arrangement, when the material passes through the crusher and then through the screen it loses height and the recirculation of the coarse fraction requires energy to return back to the crusher. Even if the conveying system does not consume energy for its operation one has to provide energy in order to elevate the circulating material back to the crusher.

Following the analysis above, it is obvious that for the optimization of the capacity of the crushing circuit one has to take into consideration the breakage rate of the material as a function of the breakage intensity, the power of the crusher motor and the height loss in the circuit. In a real plant one should also take into consideration the energy consumed for the operation of the crusher, the screen and the conveyors even if the circuit operates empty. Initially the present work assumes a system without energy consumption for the equipment when running empty in order to show the physical dimension of the problem and it will be easy for the reader to transfer the solution to an existing plant.

### **The centrifugal crusher**

A centrifugal crusher consists mainly of a rotating disc with diameter  $D=2R$ . The feed material drops at the center of the disc and is forced to rotate by two radial bars (wings) with height  $h$  starting from a distance  $R_o$  from the center of the disc up to its periphery, a length  $R$  from the center. A photo of the experimental crusher used appears in Figure 1.

Due to the rotation the feed particles are forced to the periphery by the centrifugal forces acting on them. At the edge of the periphery the particles escape from the disc having a velocity  $V$  made of two components, a peripheral velocity  $V_p$  due to the rotation and a radial centrifugal one  $V_c$  vertical to the periphery. The peripheral velocity  $V_p$  is calculated by equation (1), where  $N$  is the rotation frequency.

$$V_p = \pi \cdot D \cdot N \quad \dots(1)$$

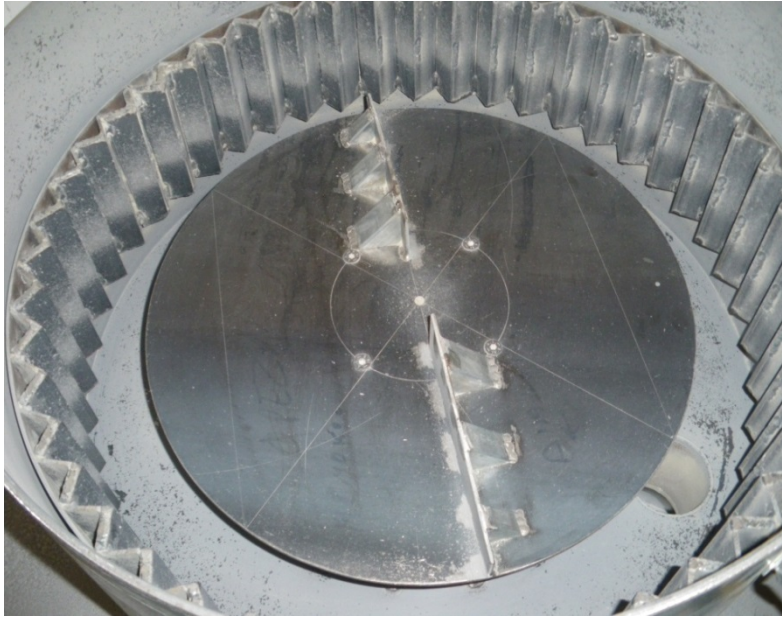


Figure 1: Inside view of the centrifugal crusher

The radial velocity  $V_c$  is perpendicular to the peripheral velocity  $V_p$  but it has the same magnitude given by eq. (1) , as is proven by D. Stamboliadis [1] and described by E. Stamboliadis et. al. [2]

The resultant  $V$  is given by equation (2) and its direction is at  $45^\circ$  to the radius because its components are equal..

$$V^2 = V_c^2 + V_p^2 \quad \dots(2)$$

The kinetic energy  $E$  of each particle is given by equation (3), where  $m$  is the mass

$$E = \frac{1}{2} \cdot m \cdot V^2 \quad \dots(3)$$

The specific energy of the particle  $e=E/m$ , energy per unit mass, also called breakage intensity, is given by equation (4), taking in to consideration eq. (1) and (2). It depends only on the diameter of the disc and the rotation frequency and it is the same for any particle regardless of mass.

$$e = \frac{E}{m} = (\pi \cdot D \cdot N)^2 \quad \dots(4)$$

The particles leaving the disc crush on the opposite wall and break into smaller ones due to the kinetic energy they acquired on the disc. The lining of the opposite wall is designed to have blades at  $45^\circ$  so that the particles strike vertical on them.

The total energy obtained by all the particles is provided by the motor that has a power  $P$ . If the feed rate of the material to the crusher is  $Q$  there is a relationship between the motor power, the through put rate of the material and the specific energy that the particles acquire before breakage and is given by equation (5).

$$P = Q \cdot e \quad \dots(5)$$

Solving equation (5) for the capacity  $Q$  taking into consideration (4) one obtains equation (6), which shows that, the throughput capacity is inversely proportional to the breakage intensity given to the particles.

$$Q = \frac{P}{e} = \frac{P}{(\pi \cdot D \cdot N)^2} \quad \dots(6)$$

Equation (6) also indicates that the throughput capacity is inversely proportional to the square of the rotation frequency that can be varied on will, while the disc diameter and the motor power are given and cannot vary for a given installation.

The residence time of a particle in the crusher is the time interval required by a particle to run the distance from  $R_o$  to  $R$ , the beginning and the end of the wings. This time interval  $t$  depends on the rotation frequency  $N$  and is given by equation (7), Manoussaki [3].

$$t = \frac{1}{2 \cdot \pi \cdot N} \cdot \ln\left(\frac{R}{R_o}\right) \quad \dots(7)$$

The maximum volume  $V_{max}$  that the material can occupy on the disc depends on the height  $h$  of the wings. The maximum quantity of material  $M_{max}$  that can be hold on the disc depends on the bulk density  $\rho$  of the material and is given by equation (8)

$$M_{max} = V_{max} \cdot \rho = \frac{\pi}{4} \cdot D^2 \cdot h \cdot \rho \quad \dots(8)$$

The percent ratio  $f$  of the mass  $M$  of the material in the crusher to  $M_{max}$  will be referred to as the saturation degree  $f = 100 \cdot M/M_{max}$ . It is obvious that the feed rate of the crusher cannot be greater than the one that gives  $f \geq 100\%$ , otherwise it will block.

### The breakage model

The specific energy, required to break a particle in the centrifugal crusher depends on its size. The particles of a particulate material are usually classified by screening in size classes and each class has an upper size  $x_1$  and a smaller size  $x_2$ . The average size  $d$  of the class is defined as  $d = \sqrt{x_1 \cdot x_2}$ . As the ratio  $x_1/x_2$  approaches to 1 the particles become isodimensional and their size tends to  $d$ .

When a size class breaks, the fraction  $F_d$  of its total mass that breaks below its lower size, depends on the breakage intensity and is given by equation (9), Stamboliadis [4]

$$F_d = \frac{e}{\Delta H_d + e} \quad \dots(9)$$

The quantity  $\Delta H_d$  is defined as the specific enthalpy required breaking a particle of size  $d$ . When the breakage intensity  $e$  provided by the crusher becomes  $e = \Delta H_d$  then  $F_d = 0,5$ . By measuring the breakage intensity that gives  $F_d = 0,5$  for several feed size classes one can find the energy size relationship. As an example the corresponding relationships for limestone and serpentine are given by equations (10) and (11) respectively, for  $d$  in mm [4].

$$\text{Limestone} \quad \Delta H_d = 1250 \cdot d^{-0.57} \quad \text{J/kg} \quad \dots(10)$$

$$\text{Serpentine} \quad \Delta H_d = 6190 \cdot d^{-1.47} \quad \text{J/kg} \quad \dots(11)$$

After this it is practically possible to calculate from equation (9) the mass fraction  $F_d$  of any size class of a material that breaks below the size class indicated by the average  $d$  for any breakage intensity  $e$ . Equation (9) is homographic and the fraction produced tends to unit as the breakage intensity increases, while equations (10) and (11) show that bigger particles break easier than smaller ones.

### The crushing circuit

Figure 2 represents a typical crushing circuit. The fresh feed is introduced into the crusher where it breaks at a fraction determined by the rotation frequency and the product goes to the screen with aperture the lower size of the fresh feed. The fraction finer than the screen aperture passes through it, while the coarser fraction is returned to the crusher, via the conveyors.

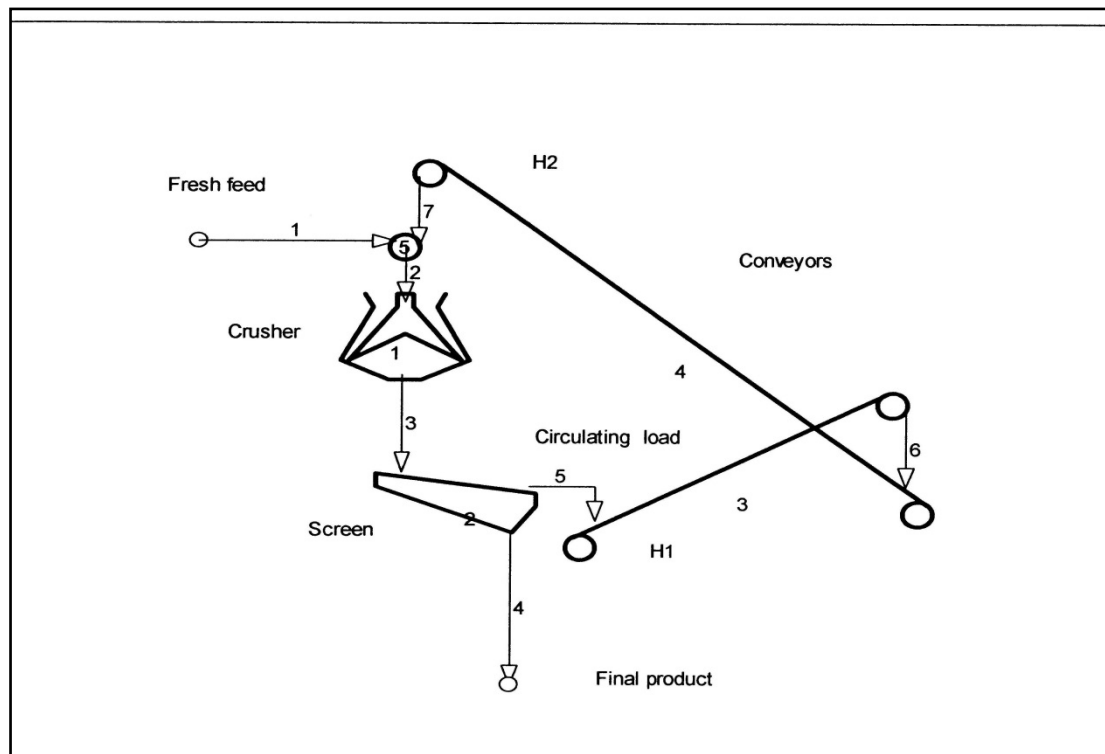


Figure 2: Crushing circuit

It is easy to see that after crushing and screening the material ends to a lower level losing height. It will take energy to lift the coarse recycled fraction from level  $H_1$  to level  $H_2$  at the feed point of the crusher. It is helpful to notice the following that hold at equilibrium.

The quantity of the final fine product equals that of the fresh feed.

The total feed to the crusher is made of the fresh feed and the recirculating coarse fraction.

According to equations (6) and (9) the total feed to the crusher increases as the rotation frequency decreases because the coarse fraction increases.

Opposite, according to the same equations, the fraction of the crusher product below the screen size is greater as the rotation frequency increases.

The energy required to lift the recirculated coarse product is higher as the breakage energy is reduced.

### A case study

It is clear from the above analysis that the effect of breakage energy has an opposite effect on the throughput capacity and on the feed fraction that breaks below size. One should be looking for an optimum rotation frequency, or the same, optimum breakage intensity, for which the production of fine material is maximum for the same total energy input. Even more find the conditions under which the energy efficiency is greater, that is the energy consumed by the crusher per ton of the final product is greater compared to the total energy consumed by the crushing circuit as a whole.

Initially let us assume that the crusher, the screen and the conveyor belts are ideal machines and consume no energy for friction when they run idle. Let us also assume the technical parameters as follows: The feed material is limestone, of size 4-5,6 mm that gives an average size  $d=4.73$  mm, a bulk density  $1400 \text{ kg/m}^3$ , and specific enthalpy  $\Delta H_d= 500 \text{ J/kg}$ , or (0.139 kWh/ton). The screen aperture is 4 mm, and the height loss is  $H_{loss}= (H_2 - H_1) = 6 \text{ m}$ . The disc diameter of the crusher is  $D=0,5 \text{ m}$ , the radius  $R=0.265 \text{ m}$ , the height of the wings  $h=0,05 \text{ m}$ , the distance of the wings from the center  $R_o=0.025 \text{ m}$  and the motor power  $P=3 \text{ kW}$ .

### The throughput of the crusher

According to equation (6) the throughput of the crusher is presented in Figure 3 and tends to infinity as the rotation frequency tends to zero.

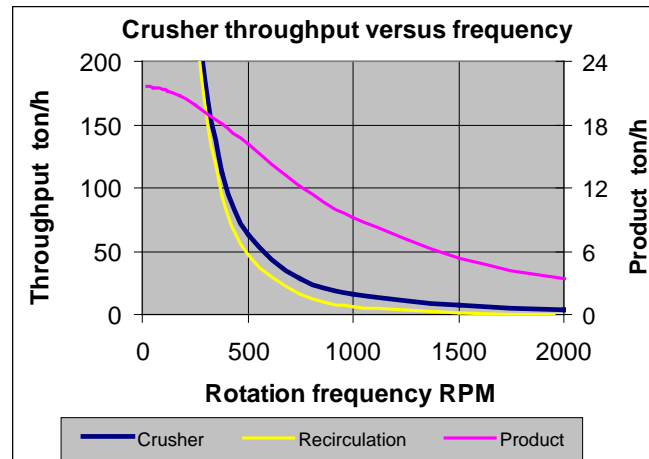


Figure 3: Throughput versus rotation frequency

The corresponding production rate  $B_d$  (right axis) of the fine material -4 mm is plotted in the same figure and is calculated from equation (12), which is the product of (6) and (9).

$$B_d = \frac{P}{\Delta H_d + (\pi \cdot D \cdot N)^2} \quad \dots(12)$$

Equation (12) tends to a maximum  $B_{dmax}=P/\Delta H_d$  as the frequency  $N$  tends to zero and is calculated to be 21,6 ton/h. The difference between the crusher throughput and the rate of final product is the circulating load that is presented in the same Figure 3. In this figure, one should take into consideration the fact that the crusher feed rate is limited by the volume saturation of the crusher that cannot be filled by more than 100%. From equations (6), (7) and (8) one has equation (13) which shows the volume saturation  $f$  that is presented in Figure 4 as a function of the rotation frequency

$$f = \frac{P \cdot \left| \ln(R/R_0) \right|}{(2 \cdot \pi \cdot R \cdot N)^3 \cdot (\pi \cdot R \cdot h \cdot \rho)} \cdot 100 \quad \dots(13)$$

In Figure 4 one can see that saturation increases as the rotation frequency decreases and for the present case it becomes 100% at a frequency 120 RPM, below which the crusher is full up and blocks. This means that Figure 3, and any other that gives a function of frequency, holds for frequencies above 120 RPM.

At this frequency although the maximum feed rate to the crusher can be 1095 ton/h the corresponding product rate is only 21.2 ton/h

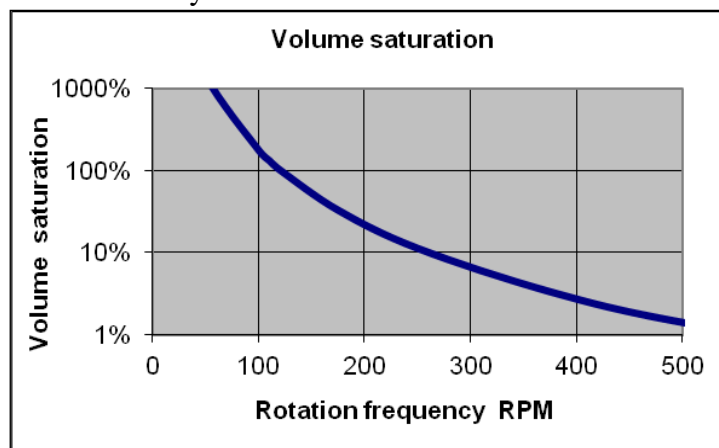


Figure 4: Volume fullness versus rotation frequency

### Circulating load

It is already obvious from Figure 3 that the circulating load increases faster than the corresponding production rate. The ratio of circulating load to the production rate is presented in Figure 5.

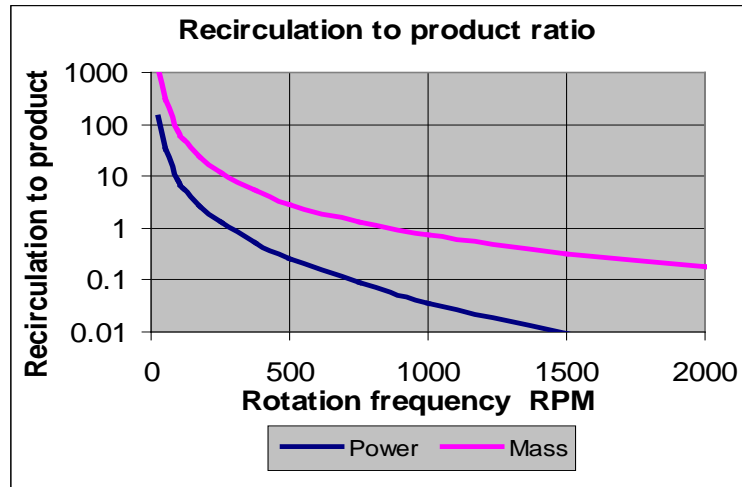


Figure 5: Circulating load to production ratio.

From this figure one can see that for the frequency 800 RPM the ratio is unit and the mass of circulating load is equal to the mass of the product. Above this frequency the circulating load is less than the product and below it increases quickly.

### Energy cost

What is interesting from the point of view of economics and environmental impact is the energy cost per unit mass of the final product. Although, the production rate increases at low frequencies and the crushing cost decreases, however due to the increase of the circulating load the final cost of the product bears the cost of the increasing circulating load as well. The energy cost per mass of the product due to the crusher and the circulation of the material is presented separately in Figure 6 together with the sum of the two that is the total cost.

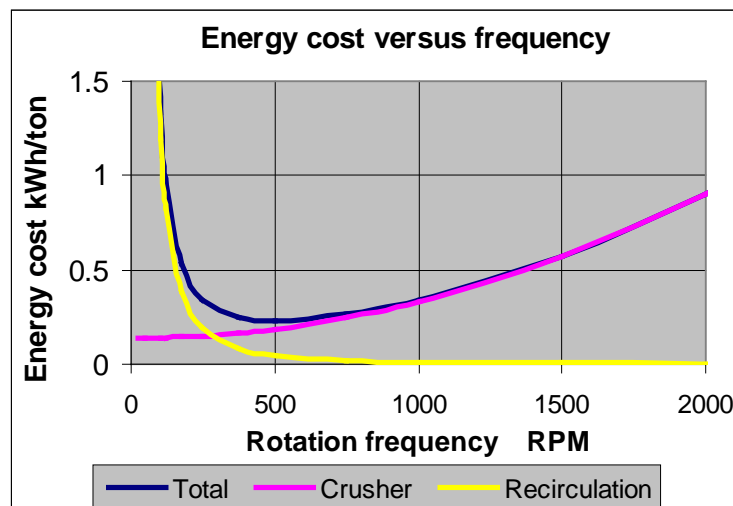


Figure 6: Energy cost versus rotation frequency

The energy cost for crushing increases with the frequency, while the cost for the circulation decreases. The sum of the two that is the final cost it has a minimum at 500 RPM, which shows the optimum conditions to run the system. The above optimum could not be derived without the knowledge and the understating of the unit process involved.

### Energy efficiency

Having defined the energy cost one can go a step further to examine the energy efficiency of the system and of the crusher as well. As explained, the energy is partly consumed for the operation of the crusher and partly for the circulation of the coarse fraction that did not break. The ratio of energy consumed for circulation to the one consumed for the crusher has been presented in Figure 5. At a frequency about 300 RPM the ratio is 1/1. At higher frequencies the system consumes more energy for crushing, while at lower ones most of the energy is consumed for the circulation.

The term *energy efficiency of the system* is defined as, the ratio of the energy actually consumed by the crusher to the total energy consumed by the system, and is presented in Figure 7 versus the rotation frequency. At high frequencies the energy is mainly consumed by the crusher and the efficiency of the system is high as well.

The term *energy efficiency of the crusher* is defined as, the ratio of the theoretical energy  $\Delta H_d$  required for crushing to the actual energy consumed by the crusher per mass of its product, and is also presented in Figure 7.

The multiplication of the energy efficiency of the system times the energy efficiency of the crusher gives the *total energy efficiency*, which is the ratio of the theoretically energy required for crushing, to overcome the coherence of the material, to the total energy consumed by the system.

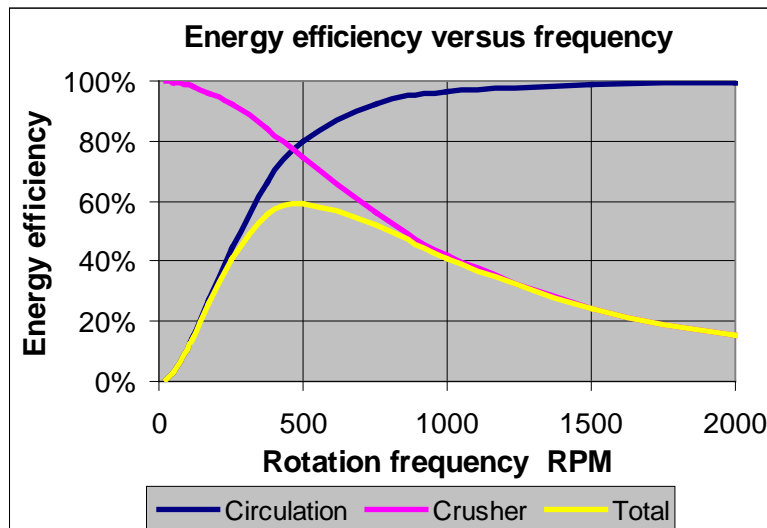


Figure 7: Energy efficiency

The total energy efficiency is also presented in Figure 7 and gives a maximum at a rotation frequency 500 RPM which is the same at which the energy cost of the system has a minimum, as shown in the previous Figure 6. The maximum of the total energy efficiency is about 60% and shows an excellent performance, in contrast to the existing impression which says that the energy efficiency of size reduction systems is far lower.

### Discussion and conclusions

An assumption made throughout the present work is that all the equipment used consume energy only for the physical work they perform and no energy is lost for friction. The energy required for crushing and circulation of the material has been calculated using this assumption. In the next step this work takes into consideration the crushing model that determines the mass fraction of the crusher feed that breaks below the desired size as a function of the rotation frequency of the crusher disc. This frequency determines the kinetic energy that the particles acquire for crushing.

For a given power of the crusher motor its throughput capacity depends on the kinetic energy given to the particle and is inversely proportional to it. The lower the energy given to



the particles, the higher the throughput capacity, up to the point the crusher blocks. However the lower the energy given to the particles the greater the mass fraction of the material that has to be recycled.

This work also indicates the optimum rotation frequency at which the final energy cost in kWh/ton of product is at a minimum, and for the case studied it is at 500 RPM. Finally the work shows that this optimum coincides with the calculated rotation frequency that gives the optimum energy efficiency. According to the crushing model the specific energy required is 0,139 kWh/ton, while the analysis of the system shows that at the optimum frequency they specific energy actually required is 0,23 kWh/ton, giving an energy efficiency 60%. The calculated efficiency 60% does not agree to the prevailing impression that crushing efficiency is one digit number %.

As already mentioned this work assumes that the equipment used are perfect and do not consume friction energy. The mechanical losses of the system are not in the scope of the present work and their calculation is left as a further work to the readers. However by experience one could say that the mechanical energy losses due to friction and the electrical energy losses due to  $\cos(\phi)$ , power factor, do not exceed 50% of the total energy. This means that even taking into consideration the mechanical losses the final energy efficiency is better than 30% relative to to calculated number 60%.

Taking the case study presented as an example one could calculate the optimum efficiency and energy cost for any closed crushing system. The only extra thing required is the mathematical model that gives the relationship between the particle size and the theoretical energy required to break it below its size class.

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